

BLACK-SCHOLES AND BEYOND OPTION PRICING MODELS

Neil A. Chriss

McGraw-Hill

New York • San Francisco • Washington, D.C. • Auckland • Bogotá • Caracas
Lisbon • London • Madrid • Mexico City • Milan • Montreal • New Delhi • San Juan
Singapore • Sydney • Tokyo • Toronto

EXHIBIT "B"

McGraw-Hill

A Division of The McGraw-Hill Companies



© Neil A. Chriss, 1997

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise, without the prior written permission of the publisher.

This publication is designed to provide accurate and authoritative information in regard to the subject matter covered. It is sold with the understanding that neither the author or the publisher is engaged in rendering legal, accounting, or other professional service. If legal advice or other expert assistance is required, the services of a competent professional person should be sought.

From a Declaration of Principles jointly adopted by a Committee of the American Bar Association and a Committee of Publishers.

Library of Congress Cataloging-in-Publication Data

Chriss, Neil.

Black-Scholes and beyond : option pricing models / Neil Chriss.
p. cm.

Includes bibliographical references and index.

ISBN 0-7863-1025-1

1. Options (Finance)—Prices—Mathematical models. I. Title.

HG6024.A3C495 1997

332.64'5—dc20

96-17361

Printed in the United States of America

6 7 8 9 0 D O 3 2 1 0 9

by the delta of the option. The conclusion to all of this is:

The theoretical value of a vanilla European call option is completely determined by its delta, the risk-free rate of interest and the time to expiration.

Summary

Let's review the main points covered so far. First of all, we have shown that the Black-Scholes formula gives rise to a hedging strategy for the short position of a vanilla European call. To show that the value of the hedging portfolio is equal to that of the option at every time, we need to: 1) know the delta of the option at every time, and 2) use the delta to determine the correct value of B_t at every time t . There is only one thing left to do: give formulas for Δ_t and B_t .

4.8 THE BLACK-SCHOLES FORMULAS FOR Δ_t AND B_t

Now that we understand the general idea of the Black-Scholes formula, let's actually see what it is. All we have to do is to give the formulas for Δ_t and B_t .

The formulas are given in terms of the cumulative normal distribution function, discussed in Chapter 2. They are:

$$\Delta_t = N(d_1), \quad d_1 = \frac{\log(S_t/K) + (r + \frac{\sigma^2}{2})(T - t)}{\sigma \sqrt{T - t}} \quad (4.8.1)$$

$$B_t = N(d_2)K, \quad d_2 = \frac{\log(S_t/K) + (r - \frac{\sigma^2}{2})(T - t)}{\sigma \sqrt{T - t}} \quad (4.8.2)$$

where

S_t = price of stock per share at time t

K = strike price

r = risk-free rate of interest

σ = volatility of stock under geometric Brownian motion model

$T - t$ = time until expiration

$N(\cdot)$ = cumulative normal distribution function

Combining this with equation (4.6.1), we present the Black-Scholes formula for vanilla European call options on a non-dividend-paying stock:

$$C_t = N(d_1) \cdot S_t - e^{-r(T-t)} K \cdot N(d_2). \quad (4.8.3)$$